Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computers & Industrial Engineering 48 (2005) 55–68
computers & industrial engineering
www.elsevier.com/locate/dsw

A cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system

Hsu-Hua Lee

Department of Industrial Engineering and Management, Ching Yun University, No. 229, Jianshing Rd., Jungli City, Taoyuan, Taiwan, ROC

Received 6 January 2004; accepted 17 July 2004
Available online 28 October 2004

Abstract

In this research, a cost/benefit model is developed for supporting investment strategies about inventory and preventive maintenance in an imperfect production system. The effect of such investments on the return is expressed as a function of measurable variables. Using this model, the decision maker can decide whether investments in inventory and preventive maintenance are necessary and how much to invest. This investment model is developed for an imperfect production system with imperfect product quality and supplied quantity. Investments in inventory and preventive maintenance increase service level for the customer and reduce the proportion of defective products, and hence affect stockout and backlog of supplied products and the delivery time to the customer. This model includes in its scope investment in inventory and preventive maintenance, manufacturing cost, inventory cost, backlog cost, stockout cost, and delay cost. This model can be used to evaluate the effects of investments on the financial cost/benefit and other relevant critical performance measures. This model can be solved by an iterative process using the Sequential Quadratic Programming Method. The optimal investment in inventory with respect to the service level and the optimal investment in preventive maintenance with respect to the proportion of defective items can be obtained first, and then other relevant costs can also be obtained.

Keywords: Inventory; Service level; Proportion of defective; Stockout; Delay cost

1. Introduction

Inventory can be used to protect the manufacturer against the randomness in production, respond to variable customer demand, and keep higher availability of goods to maintain high quality

E-mail address: hhlee@cyu.edu.tw

0360-8352/$ - see front matter
doi:10.1016/j.cie.2004.07.008

customer service. The amount of inventory needed should depend on the safety stock as so to protect against the demand uncertainty, and to achieve a high service level for satisfying customers' demand. Thus, the proper inventory level should be set based on the relationship between the investment in inventory and the service level. On the other hand, the investment in preventive maintenance will reduce the process variance and the deviation of the process mean from the target value of the measured quality characteristic, and hence reduce the proportion of defective items.

However, few efforts reported in the literature aggregately link the investment in inventory to service level and link the investment in preventive maintenance to proportion of defective items. Most of the researchers focus on a perfect manufacturing system and a perfect service level, and do not present the effect of the service level and the proportion of defective items on relevant performance measures and costs. In this research, the investment model is developed for an imperfect production system with imperfect product quality and imperfect supplied quantity. The investment in inventory increases service level for the customer, and hence affects stockout and backlog situation of supplied products. The investment in preventive maintenance reduces the proportion of defective items, and also affects the delivery time to the customer. In this paper, the investment in preventive maintenance and the investment in inventory are jointly linked to relevant performance measures related to quality, delivery time, service, inventory, and costs in an imperfect production system. The development of cost/benefit models for supporting investment strategies in inventory and preventive maintenance is crucial because it can help manufacturers in evaluating the effectiveness of their investments and in selecting optimal investment opportunities. The impact of the investment on cost/benefit should be considered and related to management performance of a company, e.g. financial performance, so that the investor can select optimally from alternative projects, including quality improvement projects, productivity improvement projects, and customer satisfaction projects. Investments in inventory and preventive maintenance should be based on their impact on quantified measures of performance, e.g. service level and quality. Therefore, what is needed is a way of expressing the quantified performance measures as a function of investments in inventory and preventive maintenance.

2. Literature review

Mehrez (1998) presented that the quality level and lot size cannot be selected optimally without considering the effect of inventory. Cheung and Leung (2000) described the relationship between quality costs of sampling and inventory in supply chain management system. Chen (2000) developed the inventory model including quality level, replenishment rate of products and the price of sales. Zeng (2001) studied the effect of the strategy of backlog on inventory cost and the loss in sales. Hayek and Salameh (2001) presented an inventory model of shortage and backlog that considers rework of defective products. Hillier (1999) studied the relationship between service level and safety stock level. Zeng and Hayya (2002) researched the inventory model with two popular service levels. Souza and Ketzenberg (2002) discussed the effect of production rate and service level on lead time in production system with rework. Gunasekaran (1995); Porteus (1986) presented investment models for reducing setup cost and increasing the quality level. Lee, Chandra, and Deleveaux (1997) studied investment in quality improvement in order to reduce the proportion of defective items and affect the inventory cost, profit loss, and internal and external failure costs.

Gupta and Campbel (1995) described an investment model that can be used to evaluate and predict the benefit of investments. Goyal and Gunasekaran (1990) also developed an investment model dealing with defective items. Leschke and Weiss (1997) studied investment models for reducing setup cost. In this research, an integrated cost/benefit model is developed; the scope includes the investment in inventory, the investment in preventive maintenance, manufacturing cost, inventory cost, backlog cost, stockout cost, and delay cost in an imperfect production system with imperfect product quality and imperfect supplied quantity. The link between the investments and relevant performance measures and costs is shown in Fig. 1. A mathematical model is developed to describe the relationships among quality, delivery time, inventory, and service, investments in preventive maintenance and inventory, and relevant costs in an imperfect production system. The hierarchical structure of investment model is unique for studying the effects of investments in inventory and preventive maintenance on service level, the proportion of defective items, and other relevant performance measures. The Sequential Quadratic Programming method can be used to solve this resource allocation problem in order to obtain optimal service level and optimal proportion of defective items. The optimal investments in inventory and preventive maintenance and the other performance measures can also be obtained.

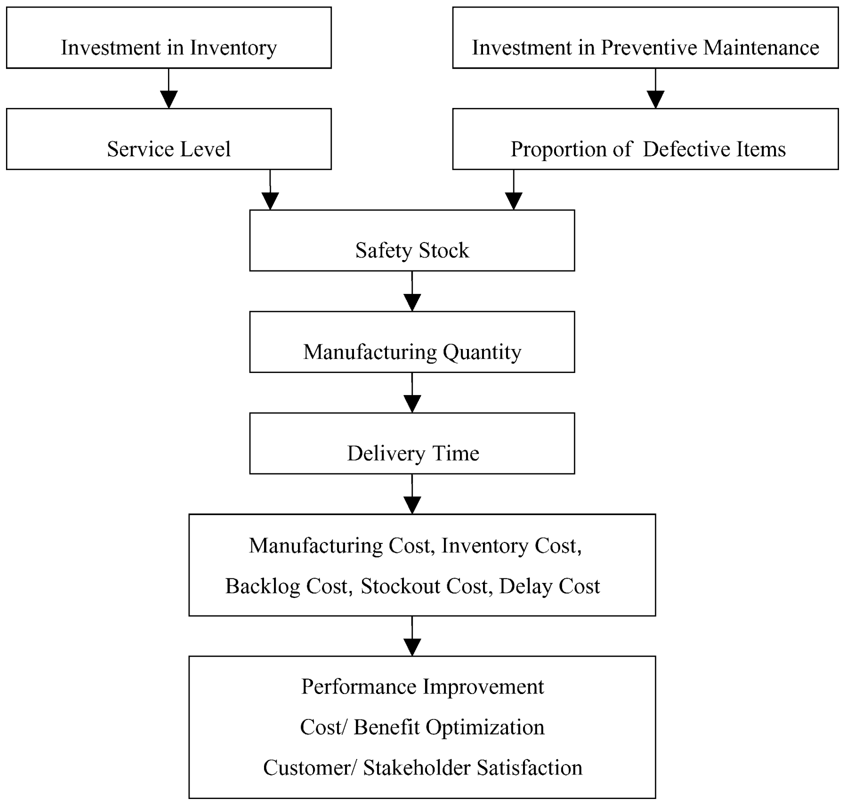

Fig. 1. An impact hierarchy for investment in inventory and investment in preventive maintenance.

3. Model development

A multi-stage manufacturing system is assumed to provide the framework for developing the analytical models. The two-tuple (i, j) indicates the jth stage of component/subassembly i, where $i=1,\ldots, N$; $j=1,\ldots, n(i)$. The production system under consideration is depicted in Fig. 2. The proportion of nonconforming and conforming components/subassemblies i at stage j can be denoted as $p(i, j)$ and $1-p(i, j)$, respectively. Let the batch quantity of finished products be Q and the service level L be defined as the proportion of demand satisfied from inventory (Zeng & Hayya, 2002).

3.1. The link between investment in inventory and service level

Hillier (1999) studied the relationship between the service level and safety stock level, but he did not consider the effect of proportion of defective items on the service level and safety stock level. In this research, the proportion of defective items and variation of demand can affect safety stock and the service level. The service level L is the proportion of finished products satisfied during the replenishment period. Let the service level at the jth stage of component/subassembly i be $L(i, j)$ and the safety stock factor be $k(i, j)$. Let the demand during production period be normally distributed, and the demand for each batch, $Q_D(i, j)$, have the mean $d(i, j)$ and variance $\sigma^2(i, j)$.

As d denotes the rate of demand, the demand rate at the jth stage of component/subassembly i without considering the variation of demand and service level can be described as

$$d(i,j) = \frac{d}{\prod_{g=j+1}^{n(i)}[1-p(i,g)]} \tag{1}$$

Let the proportion of defective items at the jth stage of component/subassembly i be $p(i, j)$. As the variation of demand and service level are not considered, the production quantity at the jth stage of

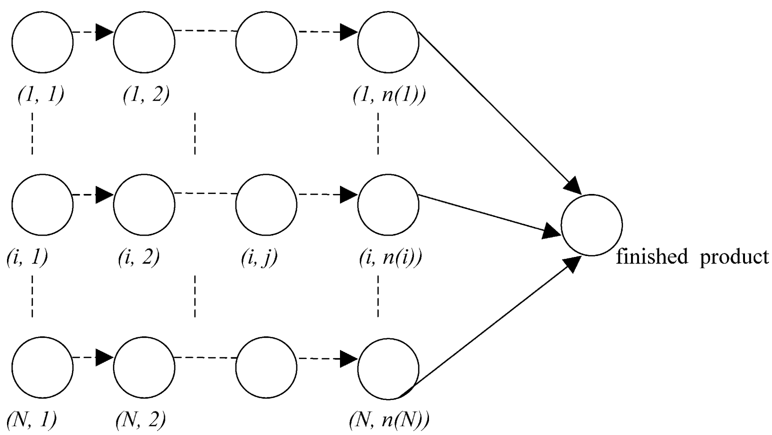

Fig. 2. Multi-stage manufacturing system chart for finished product.

component/subassembly i to satisfy the demand of batch quantity Q for finished products can be obtained

$$Q(i,j) = \frac{Q}{\prod_{g=j}^{n(i)}[1 - p(i,g)]} \tag{2}$$

Let the replenishment time be $T(i, j)$. The service level for each batch at the jth stage of component/subassembly i with considering the variation of demand and the proportion of defective items can be obtained as

$$\begin{aligned} L(i,j) &= P\{Q_{\mathrm{D}}(i,j) \le [d(i,j)T(i,j) + k(i,j)\sigma(i,j)\sqrt{T(i,j)}]\} \\ &= P\left\{ Q_{\mathrm{D}}(i,j) \le \left[\frac{Q}{\prod_{g=j}^{n(i)}[1 - p(i,g)]} + k(i,j)\sigma(i,j)\sqrt{\frac{Q}{d(i,j)\prod_{g=j}^{n(i)}[1 - p(i,g)]}} \right] \right\}, \end{aligned} \tag{3}$$

where the safety stock factor $k(i, j)$ can be obtained from standard normal distribution, and can be described as

$$k(i,j) = \Phi^{-1}[L(i,j)] \tag{4}$$

The safety stock for each batch at the jth stage of component/subassembly i can be obtained as

$$S(i,j) = k(i,j)\sigma(i,j)\sqrt{\frac{Q(i,j)}{d(i,j)\prod_{g=j}^{n(i)}[1 - p(i,g)]}} \tag{5}$$

As the cost of safety stock per unit at the jth stage of component/subassembly i is $C_{\mathrm{S}}(i, j)$, the investment in inventory at all stages for each batch can be obtained as

$$TC_{\mathrm{S}} = \sum_{i=1}^{N}\sum_{j=1}^{n(i)} C_{\mathrm{S}}(i,j)S(i,j) \tag{6}$$

3.2. Inventory cost

This cost includes all the expenses incurred because of carrying inventory. Let the production rate at the jth stage of component/subassembly i be $1/\mathrm{Pr}(i, j)$. The production time and the maximum inventory level can be denoted as $\omega(i, j)$ and $y(i, j)$, respectively. The production time, $\omega(i, j)$, can be described as (Lee, Chandra, & Deleveaux, 1997)

$$\omega(i,j) = \frac{Q[\mathrm{Pr}(i,j)]}{\prod_{g=j}^{n(i)}[1 - p(i,g)]} \tag{7}$$

The maximum inventory level at the jth stage of component/subassembly i can be obtained as

$$y(i,j) = \left\{ \frac{1}{\Pr(i,j)} - d(i,j) \right\} \omega(i,j) \tag{8}$$

As the inventory cost per unit of averaged inventory is H, the inventory cost at all stages for each batch can be obtained as

$$TC_H = \frac{H}{2} \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \left\{ \frac{1}{\Pr(i,j)} - \frac{d}{\prod_{g=j+1}^{n(i)}[1-p(i,g)]} \right\} \frac{Q[\Pr(i,j)]}{\prod_{g=j}^{n(i)}[1-p(i,g)]} \tag{9}$$

3.3. Manufacturing cost

Let the batch quantity of final products be denoted as Q. The service level of demand for final products shall be satisfied from the components/subassemblies replenished at all stages. It is assumed that the defective units are not reworked at any stage. As the proportion of defective at the jth stage of component/subassembly i is $p(i, j)$, the quantity replenished for each batch at the jth stage of component/subassembly i with considering the variation of demand and service level can be obtained as

$$M(i,j) = \frac{Q}{\prod_{g=j}^{n(i)}[1-p(i,g)]} + k(i,j)\sigma(i,j)\sqrt{\frac{Q}{d(i,j)\prod_{g=j}^{n(i)}[1-p(i,g)]}} \tag{10}$$

As the manufacturing cost per unit at the jth stage of component/subassembly i is $C_M(i, j)$, the manufacturing cost at all stages for each batch can be obtained as

$$TC_M = \sum_{i=1}^{N} \sum_{j=1}^{n(i)} C_M(i,j) M(i,j) \tag{11}$$

3.4. Stockout cost

Let the penalty for stockout at the jth stage of component/subassembly i be $A(i, j)$. The stockout cost at all stages for each batch can be obtained as

$$TC_A = \sum_{i=1}^{N} \sum_{j=1}^{n(i)} A(i,j) M(i,j) \left[\frac{1}{L(i,j)} - 1 \right] \tag{12}$$

3.5. Backlog cost

As the stockout products can be bought from the suppliers and the backlog cost at the jth stage of component/subassembly i is $B(i, j)$, the backlog cost at all stages for each batch can be obtained as

$$TC_B = \sum_{i=1}^{N} \sum_{j=1}^{n(i)} B(i,j) M(i,j) \left[\frac{1}{L(i,j)} - 1 \right] \tag{13}$$

3.6. Investment in preventive maintenance

Let the quality characteristic be $X(i, j)$ with a mean $\mu(i, j)$ and variance $\sigma_X^2(i,j)$. The target value for characteristic $X(i, j)$ is denoted as $x_0(i, j)$. The upper specification limit and lower specification limit are $\mathrm{USL}(i, j)$ and $\mathrm{LSL}(i, j)$, respectively. The investment in preventive maintenance can reduce the variance and the deviation of the mean from the target value of the quality characteristic, and hence reduce the proportion of defective items, $p(i, j)$, which can be written as

$$p(i,j) = \int_{-\infty}^{\mathrm{LSL}(i,j)} f(x(i,j))d(x(i,j)) + \int_{\mathrm{USL}(i,j)}^{\infty} f(x(i,j))d(x(i,j)) \tag{14}$$

The investment in preventive maintenance at all stages for each batch can be expressed as a function of the deviation of process mean $\mu(i, j)$ from the target value $x_0(i, j)$, and the variance for the quality characteristic $\sigma_X^2(i,j)$, and can be written as (Lee, Chandra, & Deleveaux, 1997)

$$TC_{\mathrm{PM}} = \sum_{i=1}^{N} \sum_{j=1}^{n(i)} g(\sigma_X^2(i,j), (x_0(i,j) - \mu(i,j))^2) \tag{15}$$

In practice, the investment in preventive maintenance can be written as a function of the proportion of defective items $p(i, j)$ corresponding to process mean $\mu(i, j)$ and the variance for the quality characteristic $\sigma^2(i, j)$. It is written as (Porteus, 1986)

$$TC_{\mathrm{PM}} = \sum_{i=1}^{N} \sum_{j=1}^{n(i)} [a(i,j) - b(i,j) \ln (p(i,j))], \tag{16}$$

where $a(i, j)$ and $b(i, j)$ are given constants and $a(i, j) = b(i, j) \ln (p(i, j))$ corresponding to original proportion of defective items $p(i, j)$ before improvement.

3.7. Delay cost

The production rate at the jth stage of component/subassembly i is $1/\mathrm{Pr}(i, j)$, then the total process time at the jth stage of component/subassembly i, $T_{\mathrm{p}}(i, j)$ can be written as

$$T_{\mathrm{p}}(i,j) = \left\{ \frac{Q}{\prod_{g=j}^{n(i)}[1 - p(i,g)]} + k(i,j)\sigma(i,j)\sqrt{\frac{Q}{d(i,j)\prod_{g=j}^{n(i)}[1 - p(i,g)]}} \right\} \mathrm{Pr}(i,j) \tag{17}$$

Then, the process time for component/subassembly i can be described as

$$T_{\mathrm{p}}(i) = \sum_{j=1}^{n(i)} T_{\mathrm{p}}(i,j) \tag{18}$$

Since the products are composed of all components/subassemblies, the cycle time for each batch to satisfy service level can be obtained from the maximum process time of all components/subassemblies, and can be described as

$$CT = \underset{i}{\mathrm{Max}}[T_\mathrm{p}(i)], \quad i = 1, \ldots, N \tag{19}$$

The promised delivery time for the customers for each batch to satisfied service level is T_R. As the cycle time is great than the promised time, the products will be delivered to the customers late. The penalty for delay per unit time is C_R. The delay cost for each batch to satisfy service level can be obtained as

$$TC_\mathrm{R} = C_\mathrm{R}[\mathrm{Max}\{(CT - T_\mathrm{R}), 0\}] \tag{20}$$

3.8. Total cost model

The aim is to find the optimum values of $L(i,j)$, the service level and $p(i,j)$, the proportion of defective items which can minimize the total cost, which consists of the investment in inventory, inventory cost, manufacturing cost, backlog cost, stockout cost, the investment in preventive maintenance, and delay cost. The total cost after the investments in inventory and preventive maintenance can be obtained by summing these components given in (6), (9), (11)–(13), (16), and (20). This yields

$$\begin{aligned} TC = {} & \sum_{i=1}^{N}\sum_{j=1}^{n(i)} C_\mathrm{S}(i,j)S(i,j) + \frac{H}{2} \\ & \times \sum_{i=1}^{N}\sum_{j=1}^{n(i)} \left\{ \frac{1}{\mathrm{Pr}(i,j)} - \frac{d}{\prod_{g=j+1}^{n(i)}[1 - p(i,g)]} \right\} \frac{Q[\mathrm{Pr}(i,j)]}{\prod_{g=j}^{n(i)}[1 - p(i,g)]} \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n(i)} C_\mathrm{M}(i,j)M(i,j) + \sum_{i=1}^{N}\sum_{j=1}^{n(i)} A(i,j)M(i,j)\left[\frac{1}{L(i,j)} - 1\right] \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n(i)} B(i,j)M(i,j)\left[\frac{1}{L(i,j)} - 1\right] + \sum_{i=1}^{N}\sum_{j=1}^{n(i)} [a(i,j) - b(i,j)\ln(p(i,j))] \\ & + C_\mathrm{R}[\mathrm{Max}\{(CT - T_\mathrm{R}), 0\}] \end{aligned} \tag{21}$$

3.9. Optimization

Sequential Quadratic Programming (SQP) method (Gill, Murray, & Wright, 1981; Grace & Branch, 1996) can be used to solve this resource allocation problem. The problem is transformed into an easier sub-problem which can be solved and used as the basis of an iterative process. At each major iteration an approximation is made of the Hessian of the Lagrangian function using a quasi-Newton updating method. This is then used to generate a QP sub-problem whose solution is used to form a search direction for a line search procedure. Matlab (1999 Version 5.3) is used in this research to solve the resource allocation problem based upon SQP method (Grace & Branch, 1996) with respect to the relevant decision variables including the service level and the proportion of defective items. Schittowski (1985) has implemented and tested to show that the SQP method outperforms every other tested method in terms of efficiency, accuracy, and percentage of successful solutions over a large number of test problems. The solution procedure of SQP method is as follows (Grace & Branch, 1996):

(1) At each major iteration a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated.
(2) An iterative sequence of feasible points that converge to the solution is generated.
(3) The search direction is calculated and the objective function is minimized while remaining on any active constraint boundaries.
(4) The solution to the QP subproblem produces a vector, which is used to form a new iterate. The step length parameter is determined in order to produce a sufficient decrease in a merit function.

The final investment model as per (21) can be used to predict the benefits of investment before it is made and justify investment in quality improvement projects, and can help the industries to make optimal selection of quality improvement projects for investment. The objective function can be optimized, and then the relevant investments such as the optimal investment in inventory with respect to the service level $L(i, j)$ and the optimal investment in preventive maintenance with respect to the proportion of defective items $p(i, j)$ are obtained. Other relevant costs and critical performance related to inventory, quality, delivery time, and service are also obtained.

4. Numerical example

A numerical example is presented to illustrate the model developed in this paper. The finished product is comprised of two subassemblies labeled as subassembly 1 and subassembly 2. Subassembly 1 has three stages marked as (1, 1), (1, 2), and (1, 3). Subassembly 2 has two stages marked as (2, 1) and (2, 2). The optimum values of the service level, the proportion of defective items, and the total investments and relevant costs at all stages can be obtained. The batch quantity of final products, Q, is 200. The inventory cost per unit of averaged inventory, H, is 10. The penalty for the delay per unit time, C_R, is $200,000. The promised delivery time for the customers for each batch to satisfied service level, T_R, is 1. The notations used in this example can be referred to the Appendix A. The values assumed for the original parameters are given in Tables 1 and 2. The proportion of defective items and service level are the values between zero and one. The values of $C_S(i, j)$, $C_M(i, j)$, $A(i, j)$, $B(i, j)$, $a(i, j)$, and $b(i, j)$ are given data. The original relevant costs before improvement including the investment in inventory, inventory cost,

Table 1
Original parameter values

Stage	$p(i,j)$	$L(i,j)$	$k(i,j)$	$\Pr(i,j)$	$C_S(i,j)$	$C_M(i,j)$	$A(i,j)$	$B(i,j)$	$a(i,j)$	$b(i,j)$
(1,1)	0.06	0.95	1.645	0.0025	40	40	50	70	−2813	1000
(1,2)	0.04	0.95	1.645	0.003	30	20	60	90	−3862	1200
(1,3)	0.07	0.95	1.645	0.004	40	25	70	100	−6648	2500
(2,1)	0.08	0.95	1.645	0.0028	50	30	60	80	−7577	3000
(2,2)	0.05	0.95	1.645	0.0035	60	40	70	90	−5991	2000

Table 2
Other original values

Stage	$d(i,j)$	$\sigma(i,j)$	$Q(i,j)$	$S(i,j)$	$M(i,j)$	$T_p(i,j)$	$TC_M(i,j)$	$TC_H(i,j)$
(1,1)	224	4	238	6.79	245	0.612	9803	524
(1,2)	215	4	224	6.72	230	0.692	4614	397
(1,3)	200	4	215	6.82	221	0.887	5546	215
(2,1)	210	4	228	6.86	235	0.659	7070	469
(2,2)	200	4	210	6.75	217	0.760	8691	315

Stage	$TC_S(i,j)$	$TC_A(i,j)$	$TC_B(i,j)$	$TC_{PM}(i,j)$
(1,1)	271	644	902	0
(1,2)	201	728	1092	0
(1,3)	272	817	1167	0
(2,1)	342	744	992	0
(2,2)	405	800	1029	0

manufacturing cost, backlog cost, stockout cost, the investment in preventive maintenance, and delay cost are shown in Table 3.

The relevant values and relevant cost after investments in inventory and preventive maintenance are shown in Table 4 and 5. The proportion of defective items $p(i, j)$ and the service level $L(i, j)$ after the investments in inventory and preventive maintenance are obtained. The safety stock factor $k(i, j)$ and safety stock $S(i, j)$ are obtained. The manufacturing quantity $M(i, j)$ and the processing time $T_p(i,j)$ are also reduced. The relevant costs after improvement including the investment in inventory, inventory cost, manufacturing cost $TC_M(i,j)$, backlog cost $TC_A(i,j)$, stockout cost $TC_B(i,j)$, the investment in preventive maintenance $TC_{PM}(i,j)$, and delay cost with cycle time 2.02745 as per (19) are then shown in Table 4.

The total cost including the investment in inventory, inventory cost, manufacturing cost, backlog cost, stockout cost, the investment in preventive maintenance, and delay cost can be obtained by summing these components given in (6), (9), (11), (12), (13), (16), and (20), and can be shown in Table 5.

In summary, the proportions of defective items are reduced in Table 4 and the service level is to be improved from 95% in Table 1 to 98% in Table 4 by investing \$11,334 in preventive maintenance and \$1830 in inventory investment. The resulting minimum total cost after the investment in inventory and preventive maintenance will be \$257,441. But without any investment in improvement, the original total cost is \$286,550. The investment model can be extended for flexible manufacturing environments, or logistics management system including procurement, manufacturing and delivery.

Table 3
Original relevant costs ($)

Total cost	286,550		
Investment in inventory	1493	Backlog cost	5185
Inventory cost	1922	Delay cost	238,488
Manufacturing cost	35,727	Investment in preventive maintenance	0
Stockout cost	3735		

Table 4
Relevant values after improvement

Stage	$p(i,j)$	$L(i,j)$	$k(i,j)$	$S(i,j)$	$M(i,j)$	$T_p(i,j)$	$TC_M(i,j)$	$TC_H(i,j)$
(1,1)	0.01	0.98	2.054	8.25	215	0.538	8614	504
(1,2)	0.0051	0.98	2.054	8.23	213	0.639	4265	397
(1,3)	0.0196	0.98	2.054	8.30	212	0.849	5307	204
(2,1)	0.0219	0.98	2.054	8.30	223	0.626	6706	441
(2,2)	0.05	0.98	2.054	8.43	219	0.766	8758	315

Stage	$\mathrm{TC}_S(i,j)$	$TC_A(i,j)$	$TC_B(i,j)$	$TC_{PM}(i,j)$
(1,1)	330	219	307	1792
(1,2)	247	261	391	2472
(1,3)	331	303	433	3182
(2,1)	415	273	365	3886
(2,2)	505	312	402	0

Table 5
Relevant costs after improvement ($)

Total cost	257,441		
Investment in inventory	1830	Backlog cost	1900
Inventory cost	1864	Delay cost	205,490
Manufacturing cost	33,652	Investment in preventive maintenance	11,334
Stockout cost	1371		

5. Summary

A cost/benefit model is developed for supporting investment strategies in inventory and preventive maintenance in imperfect production system in order to increase product and service quality. Investments in inventory and preventive maintenance can increase service level and reduce defects respectively, and hence affect stockout cost, backlog costs, and relevant costs in an imperfect production system with imperfect product quality and supplied quantity. In this research, analytical models are developed to quantify the effects of investment in preventive maintenance and inventory projects on tangible performance measures. Our approach is unique due to the hierarchical structure used in studying the effect of investments. The specific investment opportunities are the investment in preventive maintenance and the investment in inventory, which are at the first level. The second level includes items that are immediately affected by the investments of the fist level. These include the service level and the proportion of defective items. Safety stock is included at level three. The manufacturing quantity at level four affects the delivery time at level five. The next level includes the internal and external costs related to manufacturing cost, inventory cost, stockout cost, backlog costs, and delay cost.

The final result of this research includes an aggregate cost model that can capture the return on the investment in preventive maintenance and inventory projects in one common metric. The service level and the proportion of defective items are the decision variables that affect the amount of investments and relevant costs. The critical performance related to quality, costs, delivery time, inventory, and service are also affected. This structure enables the user to include a wider range of cost elements than traditional models. The final quality investment models are used to predict the benefits of investment before it is made and justify investment in quality improvement projects. The resource allocation model can help the industries to make optimal selection of quality improvement projects for investment. The Sequential Quadratic Programming (SQP) method can be used to solve this resource allocation problem. Some researchers have shown that the SQP method outperforms every other tested method in terms of efficiency, accuracy, and percentage of successful solutions over a large number of test problems.

Acknowledgements

The research NSC 90-2218-E-231-004 is sponsored by National Science Council.

Appendix A

The two-tuple notation (i, j) indicates the jth stage of component/subassembly i, where $i=1,\ldots,N$; $j=1,\ldots,n(i)$. The notations in this paper are defined as follows.

- $p(i, j)$ the proportion of nonconforming components/subassemblies i at stage j. (Decision variables)
- Q the batch quantity of finished products.
- L the proportion of finished products satisfied during the replenishment period.
- $L(i, j)$ the service level at the jth stage of component/subassembly i. (Decision variables)
- $k(i, j)$ the safety stock factor at the jth stage of component/subassembly i.
- $Q_D(i, j)$ the demand for each batch at the jth stage of component/subassembly i.
- $d(i, j)$ the mean of demand for each batch at the jth stage of component/subassembly i.
- $\sigma^2(i, j)$ the variance of demand for each batch at the jth stage of component/subassembly i.
- d the rate of demand of finished products.
- $Q(i, j)$ the production quantity at the jth stage of component/subassembly i without considering the variation of demand and service level.
- $T(i, j)$ the replenishment time at the jth stage of component/subassembly i.
- $S(i, j)$ the safety stock for each batch at the jth stage of component/subassembly i.
- $C_S(i, j)$ the cost of safety stock per unit at the jth stage of component/subassembly i.
- $1/\mathrm{Pr}(i, j)$ the production rate at the jth stage of component/subassembly i.
- $\omega(i, j)$ the production time at the jth stage of component/subassembly i.
- $y(i, j)$ the maximum inventory level at the jth stage of component/subassembly i.
- H the inventory cost per unit of averaged inventory.
- $M(i, j)$ the quantity replenished for each batch at the jth stage of component/subassembly i with considering the variation of demand and service level.
- $C_M(i, j)$ the manufacturing cost per unit at the jth stage of component/subassembly i.
- $A(i, j)$ the penalty for stockout at the jth stage of component/subassembly i.
- $B(i, j)$ the backlog cost at the jth stage of component/subassembly i.

$X(i, j)$ the quality characteristic at the jth stage of component/subassembly i.
$\mu(i, j)$ the mean of quality characteristic at the jth stage of component/subassembly i.
$\sigma_X^2(i, j)$ the variance of quality characteristic at the jth stage of component/subassembly i.
$x_0(i, j)$ the target of quality characteristic at the jth stage of component/subassembly i.
$\mathrm{USL}(i, j)$ the upper specification limit of quality characteristic at the jth stage of component/subassembly i.
$\mathrm{LSL}(i, j)$ the lower specification limit of quality characteristic at the jth stage of component/subassembly i.
$a(i, j)$ the constant corresponding to original proportion of defective before improvement at the jth stage of component/subassembly i.
$b(i, j)$ the constant corresponding to original proportion of defective before improvement at the jth stage of component/subassembly i.
$T_p(i, j)$ the total process time at the jth stage of component/subassembly i.
$T_p(i)$ the process time for component/subassembly i.
CT the cycle time for each batch of finished products to satisfy service level.
T_R the promised delivery time for the customers for each batch to satisfied service level.
C_R the penalty for delay per unit time.

Other notations are introduced in the text.

References

Chen, C. K. (2000). Optimal determination of quality level, selling quantity and purchasing price for intermediate firms. *Production Planning and Control*, *11*(7), 706–712.

Cheung, K. L., & Leung, K. F. (2000). Coordinating replenishments in a supply chain with quality control considerations. *Production Planning and Control*, *11*(7), 697–705.

Gill, P. E., Murray, W., & Wright, M. H. (1981). *Practical optimization*. London: Academic Press.

Goyal, S. K., & Gunasekaran, A. (1990). Effect of dynamic process quality control on the economics of production. *International Journal of Operations and Production Management*, *10*, 69–77.

Grace, A., & Branch, M. A. (1996). *Matlab optimization toolbox*. Natick, MA: The MathWorks, Inc.

Gunasekaran, A. (1995). Optimal investment and lot-sizing policies for improved productivity and quality. *International Journal of Production Research*, *33*, 261–278.

Gupta, M., & Campbel, V. S. (1995). The cost of quality. *Production and Inventory Management Journal*, *36*, 43–49.

Hayek, P. A., & Salameh, M. K. (2001). Production lot sizing with the reworking of imperfect quality items produced. *Production Planning and Control*, *12*(6), 584–590.

Hillier, M. S. (1999). Component commonality in a multiple-period inventory model with service level constraints. *International Journal of Production Research*, *37*(12), 2665–2683.

Lee, H. H., Chandra, M. J., & Deleveaux, V. J. (1997). Optimal batch size and investment in multistage production systems with scrap. *Production Planning and Control*, *8*(6), 586–596.

Leschke, J. P., & Weiss, E. N. (1997). Multi-item setup-reduction investment-allocation problem with continuous investment-cost functions. *Management Science*, *43*, 890–894.

Mehrez, O. K. A. (1998). A note on the effect of inventory costs on product quality levels. *Production Planning and Control*, *9*(6), 580–584.

Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, *34*, 137–144.

Schittowski, K. (1985). NLQPL: a FORTRAN-subroutine solving constrained nonlinear programming problems. *Operations Research*, *5*, 485–500.

Souza, G. C., & Ketzenberg, M. E. (2002). Two-stage make-to-order remanufacturing with service-level constraints. *International Journal of Production Research*, *40*(2), 477–493.

Zeng, A. Z. (2001). A partial backordering approach to inventory control. *Production Planning and Control*, *12*(7), 660–668.

Zeng, A. Z., & Hayya, J. C. (2002). The performance of two popular service measures on management effectiveness in inventory control. *International Journal of Production Economics*, *58*, 147–158.